OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

Core Mathematics 2

4722

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Tuesday 13 January 2009
Morning

Duration: 1 hour 30 minutes

4722

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

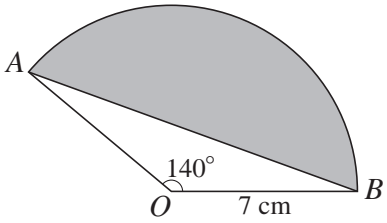
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find

(i) $\int (x^3 + 8x - 5)\,\mathrm{d}x$, **[3]**

(ii) $\int 12\sqrt{x}\,\mathrm{d}x$. **[3]**

2



The diagram shows a sector OAB of a circle, centre O and radius 7 cm. The angle AOB is 140°.

(i) Express 140° in radians, giving your answer in an exact form as simply as possible. **[2]**

(ii) Find the perimeter of the segment shaded in the diagram, giving your answer correct to 3 significant figures. **[4]**

3 A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

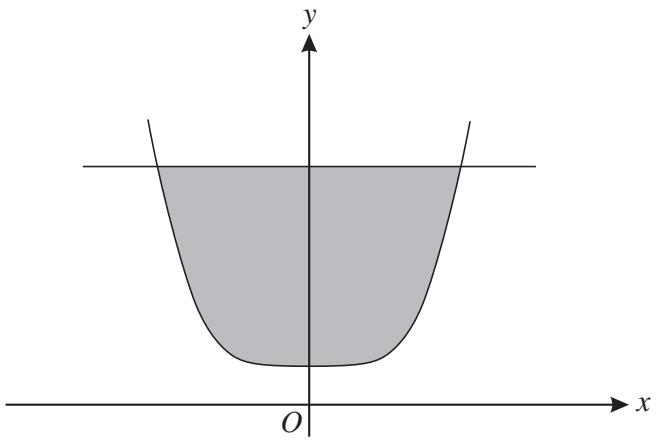
$$u_n = 24 - \tfrac{2}{3}n.$$

(i) Write down the exact values of u_1, u_2 and u_3. **[2]**

(ii) Find the value of k such that $u_k = 0$. **[2]**

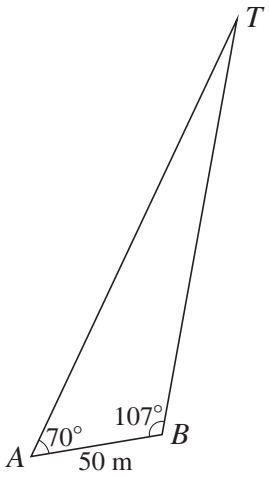
(iii) Find $\sum_{n=1}^{20} u_n$. **[3]**

4



The diagram shows the curve $y = x^4 + 3$ and the line $y = 19$ which intersect at $(-2, 19)$ and $(2, 19)$. Use integration to find the exact area of the shaded region enclosed by the curve and the line. **[7]**

5



Some walkers see a tower, T, in the distance and want to know how far away it is. They take a bearing from a point A and then walk for 50 m in a straight line before taking another bearing from a point B. They find that angle TAB is 70° and angle TBA is 107° (see diagram).

(i) Find the distance of the tower from A. **[2]**

(ii) They continue walking in the same direction for another 100 m to a point C, so that AC is 150 m. What is the distance of the tower from C? **[3]**

(iii) Find the shortest distance of the walkers from the tower as they walk from A to C. **[2]**

6 A geometric progression has first term 20 and common ratio 0.9.

(i) Find the sum to infinity. **[2]**

(ii) Find the sum of the first 30 terms. **[2]**

(iii) Use logarithms to find the smallest value of p such that the pth term is less than 0.4. **[4]**

7 In the binomial expansion of $(k + ax)^4$ the coefficient of x^2 is 24.

(i) Given that a and k are both positive, show that $ak = 2$. **[3]**

(ii) Given also that the coefficient of x in the expansion is 128, find the values of a and k. **[4]**

(iii) Hence find the coefficient of x^3 in the expansion. **[2]**

8 **(a)** Given that $\log_a x = p$ and $\log_a y = q$, express the following in terms of p and q.

(i) $\log_a(xy)$ **[1]**

(ii) $\log_a\left(\frac{a^2x^3}{y}\right)$ **[3]**

(b) **(i)** Express $\log_{10}(x^2 - 10) - \log_{10} x$ as a single logarithm. **[1]**

(ii) Hence solve the equation $\log_{10}(x^2 - 10) - \log_{10} x = 2\log_{10} 3$. **[5]**

9 **(i)** The polynomial $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = x^3 - x^2 - 3x + 3.$$

Show that $x = 1$ is a root of the equation $\mathrm{f}(x) = 0$, and hence find the other two roots. **[6]**

(ii) Hence solve the equation

$$\tan^3 x - \tan^2 x - 3\tan x + 3 = 0$$

for $0 \leqslant x \leqslant 2\pi$. Give each solution for x in an exact form. **[6]**

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1 (i)	$\int\left(x^3+8x-5\right)\mathrm{d}x=\frac{1}{4}x^4+4x^2-5x+c$	M1		Attempt integration – increase in power for at least 2 terms
		A1		Obtain at least 2 correct terms
		A1	**3**	Obtain $\frac{1}{4}x^4+4x^2-5x+c$ (and no integral sign or dx)
(ii)	$\int 12x^{\frac{1}{2}}\mathrm{d}x=8x^{\frac{3}{2}}+c$	B1		State or imply $\sqrt{x}=x^{\frac{1}{2}}$
		M1		Obtain $kx^{\frac{3}{2}}$
		A1	**3**	Obtain $8x^{\frac{3}{2}}+c$ (and no integral sign or dx) (only penalise lack of + c, or integral sign or dx once)
			6	
2 (i)	$140^{\circ}=140\times\frac{\pi}{180}$	M1		Attempt to convert 140° to radians
	$=\frac{7}{9}\pi$	A1	**2**	Obtain $\frac{7}{9}\pi$, or exact equiv
(ii)	arc $AB=7\times\frac{7}{9}\pi$	M1		Attempt arc length using $r\theta$ or equiv method
	$=17.1$	A1√		Obtain 17.1, $\frac{49}{9}\pi$ or unsimplified equiv
	chord $AB=2\times7\sin\frac{7}{18}\pi=13.2$	M1		Attempt chord using trig. or cosine or sine rules
	hence perimeter = 30.3 cm	A1	**4**	Obtain 30.3, or answer that rounds to this
			6	
3 (i)	$u_1=23^1/_3$	B1		State $u_1=23^1/_3$
	$u_2=22^2/_3$, $u_3=22$	B1	**2**	State $u_2=22^2/_3$ and $u_3=22$
(ii)	$24-{}^{2k}/_3=0$	M1		Equate u_k to 0
	$k=36$	A1	**2**	Obtain 36
(iii)	$S_{20}=\frac{20}{2}\left(2\times23\frac{1}{3}+19\times\frac{-2}{3}\right)$	M1		Attempt sum of AP with $n=20$
	$=340$	A1		Correct unsimplified S_{20}
		A1	**3**	Obtain 340
			7	
4	$\int_{-2}^{2}\left(x^4+3\right)\mathrm{d}x=\left[\frac{1}{5}x^5+3x\right]_{-2}^{2}$	M1		Attempt integration – increase of power for at least 1 term
		A1		Obtain correct $\frac{1}{5}x^5+3x$
	$=(\frac{32}{5}+6)-(\frac{-32}{5}-6)$	M1		Use limits (any two of –2, 0, 2), correct order/subtraction
	$=24\frac{4}{5}$	A1		Obtain $24\frac{4}{5}$
	area of rectangle = 19 x 4	B1		State or imply correct area of rectangle
	hence shaded area = $76-24\frac{4}{5}$	M1		Attempt correct method for shaded area
	$=51\frac{1}{5}$	A1	**7**	Obtain $51\frac{1}{5}$ aef such as 51.2, $\frac{256}{5}$
OR				
	Area = $19-(x^4+3)$	M1		Attempt subtraction, either order
	$=16-x^4$	A1		Obtain $16-x^4$ (not from $x^4+3=19$)
	$\int_{-2}^{2}\left(16-x^4\right)\mathrm{d}x=\left[16x-\frac{1}{5}x^5\right]_{-2}^{2}$	M1		Attempt integration
		A1		Obtain $\pm\left(16x-\frac{1}{5}x^5\right)$

	$= (32 - \frac{32}{5}) - (-32 - \frac{-32}{5})$	M1		Use limits – correct order / subtraction
	$= 51\frac{1}{5}$	A1		Obtain $\pm 51\frac{1}{5}$
		A1		Obtain $51\frac{1}{5}$ only, no wrong working
			7	

5 (i)	$\frac{TA}{\sin 107} = \frac{50}{\sin 3}$	M1		Attempt use of correct sine rule to find TA, or equiv
	$TA = 914$ m	A1	**2**	Obtain 914, or better
(ii)	$TC = \sqrt{914^2 + 150^2 - 2\times 914\times 150\times \cos 70}$	M1		Attempt use of correct cosine rule, or equiv, to find TC
		A1√		Correct unsimplified expression for TC, following their (i)
	$= 874$ m	A1	**3**	Obtain 874, or better
(iii)	dist from A = 914 x cos 70 = 313 m	M1		Attempt to locate point of closest approach
	beyond C, hence 874 m is shortest dist	A1	**2**	Convincing argument that the point is beyond C, or obtain 859, or better
	OR			
	perp dist = $914\times \sin 70 = 859$ m			**SR** B1 for 874 stated with no method shown
			7	

6 (i)	$S_\infty = \frac{20}{1-0.9}$	M1		Attempt use of $S_\infty = \frac{a}{1-r}$
	$= 200$	A1	**2**	Obtain 200
(ii)	$S_{30} = \frac{20(1-0.9^{30})}{1-0.9}$	M1		Attempt use of correct sum formula for a GP, with $n = 30$
	$= 192$	A1	**2**	Obtain 192, or better
(iii)	$20\times 0.9^{p-1} < 0.4$	B1		Correct $20\times 0.9^{p-1}$ seen or implied
	$0.9^{p-1} < 0.02$			
	$(p-1)\log 0.9 < \log 0.02$	M1		Link to 0.4, rearrange to $0.9^k = c$ (or >, <), introduce
	$p - 1 > \frac{\log 0.02}{\log 0.9}$			logarithms, and drop power, or equiv correct method
	$p > 38.1$	M1		Correct method for solving their (in)equation
	hence $p = 39$	A1	**4**	State 39 (not inequality), no wrong working seen
			8	

7 (i)	$6k^2a^2 = 24$	M1*		Obtain at least two of 6, k^2, a^2
	$k^2a^2 = 4$	M1dep*		Equate $6k^ma^n$ to 24
	$ak = 2$ **A.G.**	A1	**3**	Show $ak = 2$ convincingly – no errors allowed
(ii)	$4k^3a = 128$	B1		State or imply coeff of x is $4k^3a$
	$4k^3\left(\frac{2}{k}\right) = 128$	M1		Equate to 128 and attempt to eliminate a or k
	$k^2 = 16$	A1		Obtain $k = 4$
	$k = 4$, $a = \frac{1}{2}$	A1	**4**	Obtain $a = ½$
				SR B1 for $k = \pm 4$, $a = \pm\frac{1}{2}$
(iii)	$4\times 4\times \left(\frac{1}{2}\right)^3 = 2$	M1		Attempt $4\times k\times a^3$, following their a and k (allow if still in terms of a, k)
		A1	**2**	Obtain 2 (allow $2x^3$)
			9	

8 (a)(i)	$\log_a xy = p + q$	B1	**1**	State $p + q$ cwo
(ii)	$\log_a\left(\frac{a^2x^3}{y}\right) = 2 + 3p - q$	M1		Use $\log a^b = b\log a$ correctly at least once
		M1		Use $\log \frac{a}{b} = \log a - \log b$ correctly
		A1	**3**	Obtain $2 + 3p - q$
(b)(i)	$\log_{10} \frac{x^2-10}{x}$	B1	**1**	State $\log_{10} \frac{x^2-10}{x}$ (with or without base 10)
(ii)	$\log_{10} \frac{x^2-10}{x} = \log_{10} 9$	B1		State or imply that $2\log_{10} 3 = \log_{10} 3^2$
	$\frac{x^2-10}{x} = 9$	M1		Attempt correct method to remove logs
	$x^2 - 9x - 10 = 0$	A1		Obtain correct $x^2 - 9x - 10 = 0$ aef, no fractions
	$(x - 10)(x + 1) = 0$	M1		Attempt to solve three term quadratic
	$x = 10$	A1	**5**	Obtain $x = 10$ only
			10	
9 (i)	f(1) = 1 – 1 – 3 + 3 = 0 **A.G.**	B1		Confirm f(1) = 0, or division with no remainder shown, or matching coeffs with $R = 0$
	f(x) = (x – 1)(x^2 – 3)	M1		Attempt complete division by (x – 1), or equiv
		A1		Obtain $x^2 + k$
		A1		Obtain completely correct quotient (allow $x^2 + 0x - 3$)
	$x^2 = 3$	M1		Attempt to solve $x^2 = 3$
	$x = \pm\sqrt{3}$	A1	**6**	Obtain $x = \pm\sqrt{3}$ only
(ii)	$\tan x = 1, \sqrt{3}, -\sqrt{3}$	B1√		State or imply $\tan x = 1$ or $\tan x =$ at least one of their roots from (i)
	$\tan x = \sqrt{3} \Rightarrow x = {}^{\pi}/_{3}, {}^{4\pi}/_{3}$	M1		Attempt to solve $\tan x = k$ at least once
	$\tan x = -\sqrt{3} \Rightarrow x = {}^{2\pi}/_{3}, {}^{5\pi}/_{3}$	A1		Obtain at least 2 of ${}^{\pi}/_{3}, {}^{2\pi}/_{3}, {}^{4\pi}/_{3}, {}^{5\pi}/_{3}$ (allow degs/decimals)
	$\tan x = 1 \Rightarrow x = {}^{\pi}/_{4}, {}^{5\pi}/_{4}$	A1		Obtain all 4 of ${}^{\pi}/_{3}, {}^{2\pi}/_{3}, {}^{4\pi}/_{3}, {}^{5\pi}/_{3}$ (exact radians only)
		B1		Obtain ${}^{\pi}/_{4}$ (allow degs / decimals)
		B1	**6**	Obtain ${}^{5\pi}/_{4}$ (exact radians only)
				SR answer only is B1 per root, max of B4 if degs / decimals
			12	